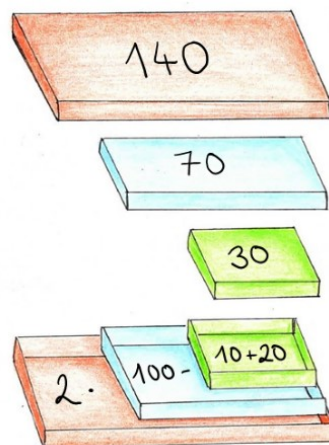



# Hiding Calculations

## (LA 10)

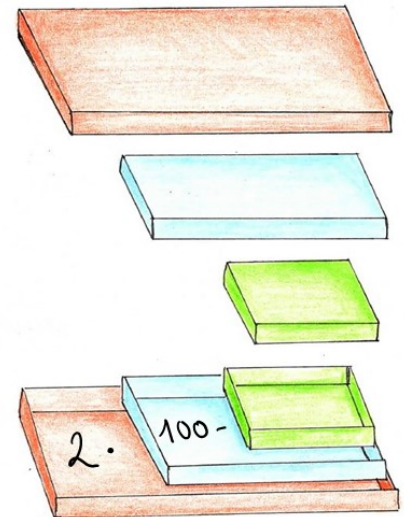



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 1. There are expressions hidden in the boxes.


- Change the term in the smallest box and calculate the answer. Write the new answer on the lid.
- Trade your boxes with another pair.
- Check the calculations of the other pair.



 2. Represent the boxes by drawing simple rectangles.


- Think up your own expressions for the boxes. Use the example on worksheet (M2) to help you write them down. You are allowed to change the terms and operations in all the boxes.
- Trade your boxes with another pair. Check each other's work.

$$2 \cdot 100 - 10 + 20 = 140$$

 3. Write down the expressions using parentheses in place of the rectangles.


- Complete the expressions on the worksheet.
- Write the equation from Task 2 with coloured parentheses on the worksheet. Work out the answer step-by-step.

$$\begin{aligned} & (2 \cdot (100 - (10 + 20))) \\ &= (2 \cdot (100 - \text{---})) \\ &= \dots \end{aligned}$$

 4. Check the validity of Mehdi's statement using two examples.



I think the parentheses are unnecessary. I get the same answer even without them.

 5. Find different calculations that work out to 100. You can have as many sets of parentheses in your equations as you want.

## 1 The Idea

In this learning arrangement, pupils explore calculating with parentheses or brackets using expressions that they find „packed“ into matchboxes and breakfast boxes. The order of operations necessary to correctly calculate expressions with multiple operations is presented and developed in a very concrete way through the „unpacking“ of the boxes.

Step-by-step the pupils are directed through Bruner’s modes of representation beginning with manipulating the expressions in boxes (enactive), progressing onto drawing rectangles containing the expressions (iconic) and finally using parentheses to write the down the expressions mathematically (symbolic), thus cementing understanding. Pupils develop an understanding of the importance of following the rules of the order of operations, namely that:

- parentheses are calculated first and are necessary
- parentheses alter the results of calculations



The calculation of expression with parentheses is done directly on terms with multiple sets of parentheses using all basic mathematical operations. This interactive (enactive) approach requires no prior knowledge on the part of the pupils regarding calculating with parentheses.

The PEMDAS/BOMDAS rule can be introduced with this learning arrangement. It is important to review and practise the order of operations in subsequent lessons and to emphasise that calculations in parentheses/brackets must be solved first.

The concept of calculating all parentheses from the inside out is used by the pupil in this learning arrangement, but not explicitly addressed. We suggest introducing this rule following completion of this learning arrangement.

This learning arrangement teaches and supports the main concepts of “*Numbers and Operations*” and “*Equations and Functions*”.

**Class 5/6**

## 2 Didactics and Teaching Methods (practical tips for teachers)

**Duration:** 2 double lessons (90 minutes each)

**Prerequisites:**

Begin by reviewing the sequence rules (PEMDAS/BOMDAS) in the order of operations: multiplication and division before addition and subtraction, as well as working from left to right. The use of parentheses does not have to have been previously covered.

For this learning arrangement, each pair needs two breakfast boxes of differing sizes that can fit into one another. The pupils need to bring these to the lessons. A set of boxes must be prepared for the introduction and each pair group also needs a matchbox.

**Introduction:**

The teacher demonstrates the calculation of the expression from Task 1

$(2 \times (100 - (10 + 20))) = 140$  with the prepared boxes. First, show the pupils the largest box labelled with 140 and ask what can be inside, creating a bit of mystery and sparking the pupils curiosity. The boxes are then unpacked one after the other, revealing the expressions. It is advisable to articulate what is being calculated: „I’ve doubled the number.“ or „I’ve subtracted the number from 100.“

Following this demonstration, the pupils construct their box calculations in pairs. Each pair needs a matchbox, three strips of wider masking tape and a felt tip pen. They label their boxes like the example. Worksheet ([M1](#)) can be passed out as a guide. Should there be pupils in the class who cannot yet calculate numbers up to 1000, then a simplification of the expressions, for example  $(2 \times (10 - (1 + 2)))$  can be useful.

**Nr 1.:**

Task 1a) was chosen specifically because it is accessible for all pupils. It allows the function of the boxes to be worked out within a controlled framework. Mathematically gifted pupils can be challenged to change more than simply the expression in the smallest box. The extension task to Nr 1 ([M3](#)) can be handed out for more practice.

Pupils work hands-on in Task 1 and do not need to record their results. Their work is checked by trading with a partner group. The extension task is also not written down.

Work on Task 1 should be completed by all pupils at the same time so that Task 2 can be explained in a whole class discussion. Use of the extension activities ([M3](#)) for quicker workers makes this possible.

**Nr 2.:**

The calculations with the boxes are now written down with the help of rectangular frames. This extra step aids in comprehension and helps pave the way for the next step: the abstract use of parentheses. For starters, only the left side of the worksheet ([M2](#)) is to be filled in. Have the pupils fold the worksheet in half.

Using colours makes it easier to keep track of the calculations, especially for weaker pupils, and is also helpful when working on Task 3.

Quicker, more talented pupils can write down additional examples in their exercise books. Use the card ([M3](#)) with extension tasks for this purpose.

**Nr 3.:**

This is the first time that the pupils are confronted with written expressions using parentheses. The equations from Task 2 are now to be written using correct mathematical notation. The first example has been simplified by the presented framework. The children should use this framework to complete the rest of the equations.

Differentiation occurs for more talented pupils as they write down their extension equations from Task 2, which do not necessarily follow the same pattern. Highly gifted pupils can be challenged to produce calculations without the help of the boxes.

**Nr 4.:**

Pupils have worked out calculating with parentheses in Tasks 1 through 3. By having to prove their answers using their own work in Task 4, the pupils should come to the realisation that parentheses can change the results of their calculations. The P in PEMDAS comes first for a reason! Tip cards can be used here if necessary.

The outermost parentheses are unnecessary, meaning that the outer box does not change the answer. It is also possible, that the inner parentheses also do not affect the answer. The original example has been so chosen because all the inner parentheses are vital to calculating the correct answer.

**Nr 5.:**

This open-ended task allows for more practice in using parentheses to solve equations. Differentiation occurs naturally as pupils are able to individually decide how many sets of parentheses and which terms they use to calculate 100.

**3 References to the Berlin State Curriculum**

3.1 Process oriented mathematical standards of this learning arrangement <sup>1</sup>

Process oriented mathematical competencies	The pupils can
<b>argue mathematically</b>	<ul style="list-style-type: none"> <li>recognize correlations and structures and pose hypotheses to mathematical situations</li> <li>find examples and counter-examples for mathematical statements</li> <li>question mathematical statements and check their correctness</li> </ul>
<b>solve problems mathematically</b>	<ul style="list-style-type: none"> <li>tackle problems for which they have not yet learned a formal routine solution (use alternate strategies to help themselves)</li> <li>use mathematical knowledge, skills and competencies to work on problems</li> </ul>
<b>use mathematical representations</b>	<ul style="list-style-type: none"> <li>choose, use and develop appropriate representations for working mathematically on real world situations and problems</li> <li>change one representation to another</li> <li>compare different representations</li> </ul>
<b>use symbols, formal and technical elements</b>	<ul style="list-style-type: none"> <li>use tables, terms, equations and diagrams to describe facts and situations</li> <li>translate symbolic and formal language into natural language and vice versa</li> <li>use a checking method</li> </ul>
<b>communicate mathematically</b>	<ul style="list-style-type: none"> <li>describe their approach, understand the approaches of others and reflect on common approaches</li> <li>work on tasks in a group</li> </ul>

3.2 Content-related mathematical competencies of this learning arrangement <sup>2</sup>

<sup>1</sup> vgl. Rahmenlehrplan Jahrgangsstufen 1-10, Teil C Mathematik, S. 19-21, Berlin, Potsdam 2015

<sup>2</sup> vgl. Rahmenlehrplan Jahrgangsstufen 1-10, Teil C Mathematik, S. 22-31, Berlin, Potsdam 2015



## B Information for Teachers: Hiding Calculations (Theme Box: Equations and Functions – LA 10/H)

Theme	Competency	Level
<b>Numbers and Operations</b>	The pupils can <ul style="list-style-type: none"> <li>appropriately use strategies, methods and rules of calculations and the laws of the basic operations in the range of natural numbers up to a million</li> </ul>	C
<b>Equations and Functions</b>	The pupils can <ul style="list-style-type: none"> <li>represent terms and equations (even with multiple operations)</li> </ul>	C

### 3.3 Themes and Content of this learning arrangement<sup>3</sup>

Theme	Content	Niveau
<b>Numbers and Operations</b>	The pupils <ul style="list-style-type: none"> <li>link multiple operations in expressions following the rules of the order of operations PEMDAS (specifically multiplication and division before addition and subtraction and parentheses first) with natural numbers</li> </ul>	C
<b>Equations and Functions</b>	The pupils <ul style="list-style-type: none"> <li>represent real world situations using terms and equations (including those with several mathematical operations)</li> </ul>	C
	The pupils <ul style="list-style-type: none"> <li>independently check the correctness of an answer</li> </ul>	C

### 3.4 References to the general curriculum for language development<sup>4</sup>

<b>Standards of the g.c. language development</b>	The pupils can
<b>Production/ Speaking</b>	<ul style="list-style-type: none"> <li>recount observations</li> <li>express their own thoughts on a situation or text</li> <li>articulate and justify hypotheses</li> </ul>

<sup>3</sup> vgl. Rahmenlehrplan Jahrgangsstufen 1-10, Teil C Mathematik, S. 31ff, Berlin, Potsdam 2015

<sup>4</sup> vgl. Rahmenlehrplan Jahrgangsstufen 1-10, Teil B Fachübergreifende Kompetenzentwicklung, S. 6-10, Berlin, Potsdam 2015

## 4 Language Development

### 4.1 Possible language difficulties in task directions

Task	Original text	Alternatives
3	Complete the calcutions on the worksheet.	Fill in the blanks in the equations on the worksheet.
4	Check the validity of Mehdi's statement using two examples.	Is Mehdi's statement correct? Find two examples.

*Pupils must understand the following vocabulary:*  
 box, matchbox, frame, hide, check, rectangle

### 4.2 Vocabulary List for Comprehension

*The teacher must be sure that the pupils understand the following mathematical vocabulary.*

Nouns	Verbs	Adjectives / Aderbs
number parentheses step equation term expression answer rectangle	represent calculate double check	step-by-step simplified

### 4.3 Subject relevant vocabulary and theme specific phrases

During the course of this learning arrangement, the pupils will actively use the following vocabulary and phrases. These will be the foundation for establishing a relevant word list to use when presenting their results.

There is an expression / term hidden in each box.

On the lid is the answer.

Each box represents a set of parentheses.

The frames stand for parentheses.

Operations in parenthesis are always calculated first.

I always solve the operation in the inner set of parentheses for the term first. / The answer of the operation in the inner set of parentheses must always be calculated first.

If I change the number/ the term/ the operation in the smallest box, then the answer will also change.

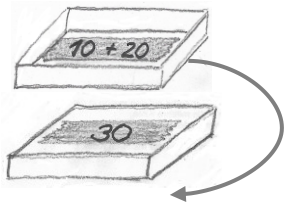
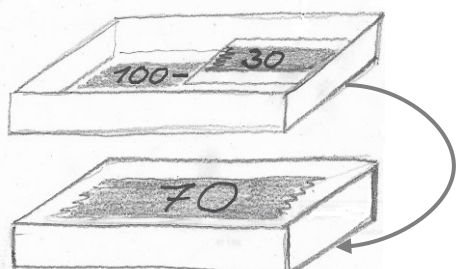
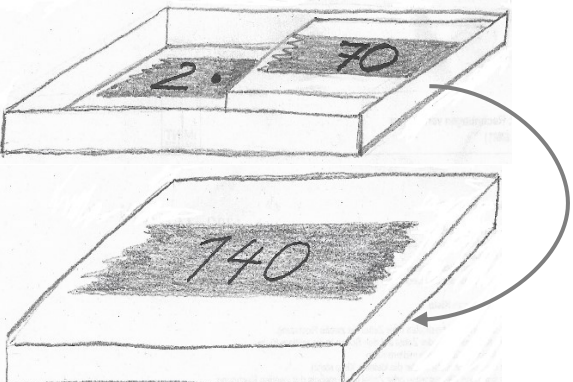
If the number / term/ operation in the smallest box is changed, it affects the final answer.

**5 Materials needed for this learning arrangement**

Phase / Task	Amount	Materials	Preparations
Introduction	1 demo set for the teacher	box model	matchbox (empty), 2 breakfast boxes that fit inside each other (larger than the matchbox), masking tape (wide), felt tip marker
for all tasks in the LA	1 per pair	Task Sheet ( <a href="#">LA</a> )	copy or project onto digital whiteboard
Task 1	1 per pair	Instructions for box model ( <a href="#">M1</a> )	copy or project onto digital whiteboard
	1 per pair	box model	see above
	1 per pair (if needed)	Extension activities for Task 1 ( <a href="#">M3</a> )	copy
Task 2	1 per pupil	Worksheet ( <a href="#">M2</a> )	copy
	1 per pair (individuell)	Extension activities for Task 2 ( <a href="#">M3</a> )	copy exercise book or paper
Task 3	1 per pupil	Worksheet ( <a href="#">M2</a> )	passed out during Task 2
	1 per pair (if needed)	Extension activities for Task 3 ( <a href="#">M3</a> )	copy
Task 4	1 per pair (if needed)	Tip cards ( <a href="#">M4</a> )	copy
All tasks in this LA	as needed	Vocabulary Cards for a Word Wall ( <a href="#">M5</a> )	enlarged, laminated copies or project onto digital whiteboard

**Instructions for making the box model for the introduction**

Task: Put together the box model with a partner. You will receive a matchbox and need to label it as pictured here, along with the boxes you brought and then put them inside each other.

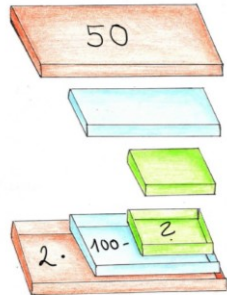
<p><b>Labelling the matchbox</b></p> <ol style="list-style-type: none"> <li>1. Put a strip of masking tape on the bottom of the inside of the matchbox.</li> <li>2. Write the term <b>10 + 20</b> on the tape.</li> <li>3. Close the matchbox.</li> <li>4. Put a strip of masking tape on the lid.</li> <li>5. Write the answer <b>30</b> on the strip of tape.</li> </ol>	
<p><b>Labelling the middle-sized box</b></p> <ol style="list-style-type: none"> <li>1. Put a strip of masking tape on the bottom of the inside of the middle-sized box.</li> <li>2. Put the matchbox inside the middle-sized box on the right side.</li> <li>3. Write the expression <b>100 -</b> to the left of the matchbox.</li> <li>4. Close the box.</li> <li>5. Put a strip of masking tape on the lid.</li> <li>6. Write the answer <b>70</b> on the strip of tape.</li> </ol>	
<p><b>Labelling the largest box</b></p> <ol style="list-style-type: none"> <li>1. Put a strip of masking tape on the bottom of the inside of the largest box.</li> <li>2. Put the middle-sized box inside the largest box on the right side.</li> <li>3. Write the expression <b>2x</b> to the left of the middle-sized box.</li> <li>4. Close the box.</li> <li>5. Put a strip of masking tape on the lid.</li> <li>6. Write the answer <b>140</b> on the strip of tape.</li> </ol>	

**C Material: Hiding Calculations**  
**(Theme Box: Equations and Functions – LA 10/M2)**

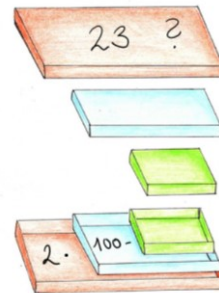
Task 2	checked	Task 3
$2 \cdot 100 - 10 + 20 = 140$		$\begin{aligned} & \left( 2 \cdot \left( 100 - (10 + 20) \right) \right) \\ & = \left( 2 \cdot \left( 100 - \underline{\hspace{1cm}} \right) \right) \\ & = \left( 2 \cdot \underline{\hspace{1cm}} \right) \\ & = \underline{\hspace{1cm}} \end{aligned}$

## Extension Activities for Task 1

- a) Change the operation in the smallest box so that the final answer is 50.



- b) Is it possible to have the answer 23 on the lid of the largest box? Justify your answer.

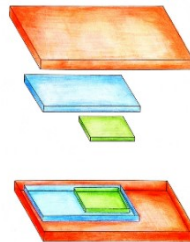


- c) Is a result of 23 on the lid of the largest box possible, if you can change the operations in all of the boxes?

## Extension Activities for Task 2

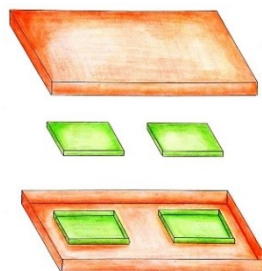
- a) Change the placement of your boxes and come up with new calculations. Write down the equations in your exercise book.

example



- b) Use different boxes to create new calculations. Write down the answers in your exercise book.

example



LA 10 / Task 2

## Extension Activities for Task 3

- Come up with an expression with parentheses.
- Calculate the expression step-by-step without using boxes.
- Check your answer, if possible, by putting together a box model.

Tip Cards

**for Task 4**

**Tip 1:**

Write down the expression  $(2 \cdot (100 - (10 + 20)))$  with  
and without parentheses.

Calculate both terms and compare your answers.



LA 10 / Task 4

**For Task 4**

**Tip 2:**

Calculation without parentheses

$$\begin{aligned} & 2 \cdot 100 - 10 + 20 \\ & = 200 - 10 + 20 \\ & = 190 + 20 \\ & = \underline{\quad} \end{aligned}$$



LA 10 / Task 4

**For Task 4**

**Tip 3:**

Choose another example and calculate the answer with and  
without parentheses.



LA 10 / Task 4

Vocabulary Card for a Word Wall

term

$$3 \cdot (4 + 2)$$

calculation

result / answer

box

lid

frame

parentheses

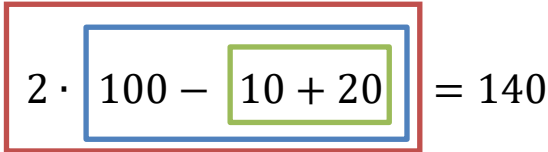
inner / outer  
brackets

**Solutions to the learning arrangement**

1. Examples of individual pupil work:

The original equation was  $(2 \cdot (100 - (10 + 20))) = 140$ . The term in the smallest box is changed from  $10 + 20$  to  $10 + 10$ . The new equation is:  $(2 \cdot (100 - (10 + 10))) = 160$

2. and 3.

Task 2	Task 3
	$\begin{aligned} & (2 \cdot (100 - (10 + 20))) \\ &= (2 \cdot (100 - 30)) \\ &= (2 \cdot 70) \\ &= 140 \end{aligned}$
varied, individual answers	

4. example solution

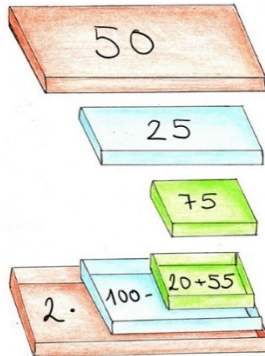
Example (without parentheses)	Example (leaving out middle set of parentheses)	Example (not using outer set of parentheses)
$\begin{aligned} & 2 \cdot 100 - 10 + 20 \\ &= 200 - 10 + 20 \\ &= 190 + 20 \\ &= 210 \end{aligned}$	$\begin{aligned} & (2 \cdot 100 - (10 + 20)) \\ &= (200 - (10 + 20)) \\ &= (200 - 30) \\ &= 170 \end{aligned}$	$\begin{aligned} & 2 \cdot (100 - (10 + 20)) \\ &= 2 \cdot (100 - 30) \\ &= 2 \cdot 70 \\ &= 140 \end{aligned}$
<p>The answer changes because the order of operations is changed.</p>	<p>The answer changes because the order of operations is changed.</p>	<p>The answer is the same. This means that the outer set of parentheses are unnecessary and do not affect the outcome. This means you do not need the last box.</p>

5. varied, individual answers

## Answers to Extension Activities

### Extension Activity 1

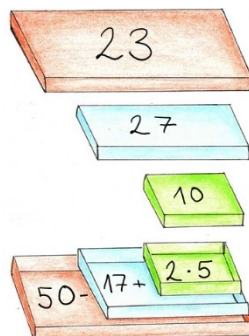
- a) Varied answers. The expression in the smallest box has to result in 75 in order for the final answer to be 50.



- b) No, it is not possible. Because the final calculation is doubling ( $2 \cdot \underline{\quad}$ ), the final answer will always be an even number.

- c) If you can change the operations and terms in all the boxes, then a final answer of 23 is possible.

Example solution:



### Extension Activity 2

- a) individual, varied answers.  
b) individual, varied answers.

### Extension Activity 3

individual, varied answers.

## Sources

Senatsverwaltung für Bildung, Jugend und Familie (Hrsg.) (2015): Rahmenlehrplan Jahrgangsstufen 1-10, Teil B. Fachübergreifende Kompetenzentwicklung, Berlin, Potsdam 2015

Senatsverwaltung für Bildung, Jugend und Familie (Hrsg.) (2015): Rahmenlehrplan Jahrgangsstufen 1-10, Teil C. Mathematik, Berlin, Potsdam 2015

## Further Reading:

Steinweg, Anna Susanne (2013): Algebra in der Grundschule. Berlin, Heidelberg: Springer

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2	Mehdi	Mehdi, Laura Jahn, <a href="#">CC BY SA 4.0</a> , LU Würfel kippen
2	symbols for individual, partner and group work	Symbole „Einzelarbeit“, „Partnerarbeit“, „Gruppenarbeit“, Solveg Schlinske, <a href="#">CC BY SA 4.0</a> , LU Sehenswürdigkeiten
3	Expression in breakfast boxes	Rechnungen in Frühstückskisten, erstellt von Mia Huber, <a href="#">CC BY SA 4.0</a> , LU Rechnungen verstecken
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