

Multiplication and the Rectangle Method

(LA 2)

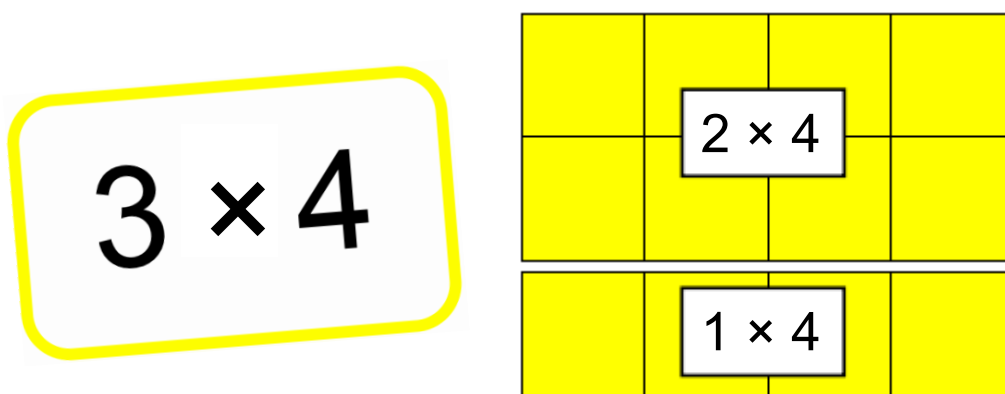
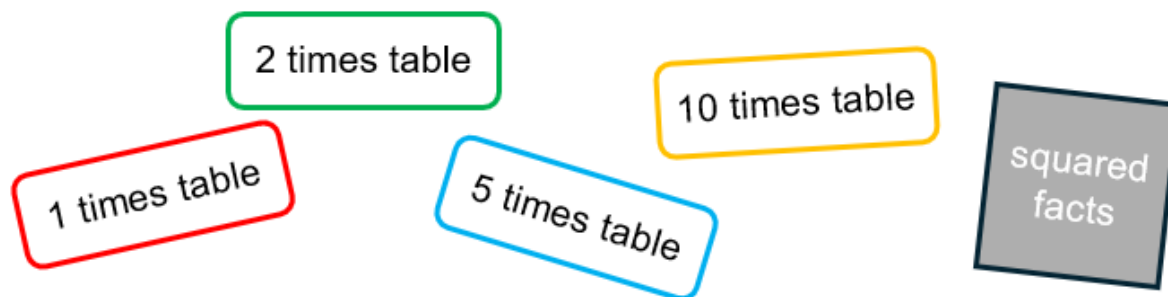




Table of Contents

A Learning Arrangement	2
B Information for Teachers (with references to the Berlin State Curriculum including tips for language development specific to this learning arrangement)	3
C Worksheets / Materials / Language Support / Answer Keys	10
D Appendix	22



Multiplication with Basic Facts

-  1. a) Cover your multiplication fact task card with the basic fact rectangles (arrays).
b) Draw the solution on graph paper.
Write the basic facts in the rectangles (arrays).
-  2. Trade the task cards in your group.
Can you use different basic fact arrays to cover the task card? Draw.
3. Make a poster for your times table.
 - Cut out your solutions.
 - Did you find a breakdown into basic facts for each multiplication fact?
 - Decide on an arrangement for your solutions and glue them onto your poster.
4. Is Benjamin correct? Support your answer.



Each of the multiplication facts can be broken down with only two basic facts.

1 The Idea

Every multiplication fact of the 1-10 times tables can be broken down (using the commutative and distributive properties) with the help of the core multiplication facts (1s, 2s, 5s and 10s times tables) and the squared facts. The core multiplication facts are usually easy for children to understand and remember. The squared facts are also familiar or can be quickly learned. These can then be used to understand and learn the remaining (sometimes more difficult) multiplication facts of the 1-10 times tables. A prerequisite is that all the core multiplication facts have been memorised.

At the same time, the principle of multiplication as repeated addition, or the adding of equal groups, is visually presented using the arrays in this rectangle method. The work with these rectangular fields enables the long-term development of the basic concept of multiplication and allows for connections to mathematical instruction in upper levels including double digit multiplication, surface area calculations, the multiplication of fractions and decimal fractions, and binomial equations.

The children work with prepared basic fact rectangles (arrays) to lay out different solutions to various multiplication table facts. They break down these facts, draw the solutions, and transpose them during this learning arrangement. This shift between the different modes of cognitive representation (enactive, iconic, symbolic) helps cement children's understanding of the process of multiplication.

The colours of the times tables used in this learning arrangement are based on those of Maria Montessori: 3 times tables: pink, 4 times tables: yellow, 6 times tables: purple, 7 times tables: white, 8 times tables: brown, 9 times tables: dark blue.

This learning arrangement teaches and develops the ideas and competencies found in the mathematical themes of the Berlin State Curriculum [L1] *Zahlen und Operationen* (Numbers and Operations) and [L4] *Gleichungen und Funktionen* (Equations and Functions)

Class 2/3

2 Didactics and Teaching Methods (practical tips for teachers)

Duration: 4 lessons (45 minutes each)

Prerequisites/Preparation:

The core multiplication facts (1, 2, 5 and 10 times tables) and the squared facts have been worked on and memorised. The children are familiar with the presentation of multiplication facts as arrays (rectangles). They have a basic understanding of addition (here: putting together the arrays) and subtraction (here: covering up arrays).

The [multiplication fact task cards](#) and the [basic fact rectangles \(arrays\)](#) (extra materials) have been laminated and exactly cut out by the teacher beforehand.

Introduction:

The teacher presents the times tables chart ([M1](#)) with all the 100 facts of the 1-10 times tables. The already learned rows (1, 2, 5, and 10 times tables) are covered. The pupils are split up into six groups. Each group gets the multiplication fact cards for one of the remaining rows on the

times tables chart and sorts them into „known“ and „unknown“. As a class, the „known“ are identified as swaps of core facts and the squared facts ([M5](#)) and are laid aside and covered on the chart.

The teacher makes the statement that all the „unknown“ multiplication facts left on the chart (five facts per row) can all be calculated with the help of already known facts and therefore do not have to be memorised.

The learning arrangement worksheet ([LA](#)) is passed out to the pupils or projected.

Nr 1.:

The pupils in each group divide up the multiplication fact task cards amongst themselves from the row they have been assigned to investigate. They individually use the core multiplication fact arrays to cover their task cards rectangles and then draw the breakdown on graph paper ([M2](#)). They label their drawings by writing the multiplication fact and the core facts used in the breakdown. Pupils can draw with rulers or trace around the laminated core fact arrays. The tip card ([M3](#)) can be made available.

If the question comes up, why some of the core multiplication fact arrays are grey, the teacher should challenge the pupils to look carefully at these arrays and think about when the use of these rectangles is advantageous (at the latest, in Task Nr 4).

Nr 2.:

The pupils trade task cards within their groups and their breakdowns and try to find different arrays that would also solve the problems.

Nr 3.:

Each group makes a poster for the times table row they examined. The 9 times table group will need a larger piece of paper. The pupils cut out their solutions from the graph paper and decide on a clear arrangement of all the possibilities. For each multiplication fact, every drawn solution is clearly arranged and glued onto the poster with the breakdown in core facts also written down. ([Example](#)).

Nr 4.:

This challenging task gives pupils the chance to discover the subtraction of core facts as a useful possibility to calculate multiplication facts. The tip cards, which provide the crucial hint, can be used ([M4](#)).

Conclusion:

The posters and findings from tasks 3 and 4 are discussed as a class. By examining the posters, the pupils realise that all the multiplication facts can be derived from the core facts. Particularly creative breakdowns should be highlighted and the use of subtraction (especially in the 8 and 9 times tables) should be emphasised as a possibility (for example: $8 \times 3 = 10 \times 3 - 2 \times 3$).

3 References to Berlin State Curriculum

3.1 Process oriented mathematical standards of this learning arrangement¹

Process oriented mathematical competencies	The pupils can
argue mathematically	<ul style="list-style-type: none"> question mathematical statements and check their correctness evaluate results and the context of application
solve problems mathematically	<ul style="list-style-type: none"> tackle problems for which they have not yet learned a formal routine solution (use alternate strategies to help themselves) develop and use solution strategies (for example from guessing to systematic trial and error)
use mathematical representation	<ul style="list-style-type: none"> change one representation into another alternate between different levels of representation (translation)
use symbols, formal and technical elements	<ul style="list-style-type: none"> use formal calculation strategies (quick mental arithmetic and automated procedures)
communicate mathematically	<ul style="list-style-type: none"> describe their approach, understand the approaches of others and reflect on common approaches use mathematical vocabulary and symbols appropriately when describing and documenting solutions work on tasks in a group

3.2 Content-related mathematical competencies of this learning arrangement²

Theme	Competency	Level
Numbers and Operations	The pupils can <ul style="list-style-type: none"> describe connections between the four basic mathematical operations in a range of natural numbers up to 100 use calculation strategies, methods, rules and laws of basic arithmetic appropriately on natural numbers 	B
Equations and Functions	The pupils can <ul style="list-style-type: none"> represent terms and equations with a basic operation solve simple equations with a basic operation 	B

¹ vgl. Rahmenlehrplan Jahrgangsstufen 1-10, Teil C Mathematik, S. 19-21, Berlin, Potsdam 2015

² vgl. Rahmenlehrplan Jahrgangsstufen 1-10, Teil C Mathematik, S. 22-31, Berlin, Potsdam 2015

3.3 Themes and Content of this learning arrangement³

Theme	Content	Niveau
Numbers and Operations	<p>The pupils</p> <ul style="list-style-type: none"> • calculate products using memorised core facts • develop ideas about the basic mathematical operations in dynamic and static: <ul style="list-style-type: none"> – addition (add to, increase)⁴ – subtraction (take away, difference) – multiplication (repeated addition of equal groups, recognising multiplicative structures) • alternate between story problems, notation, calculation und pictorial representation of the basic mathematical operations in the range of natural numbers to 100 • describe connections between the four basic mathematical operations in a range of natural numbers up to 100 	B
Equations and Functions	<p>The pupils</p> <ul style="list-style-type: none"> • make up mathematical story problems and draw pictures relating to given terms and equations (with operations) • find numerical terms with the same value (for example through the break down of numbers) 	B

3.4 References to the general curriculum for language development⁵

Standards of the general language learning curriculum	The pupils can
Awareness	<ul style="list-style-type: none"> • adequately use subject specific vocabulary
Interaction	<ul style="list-style-type: none"> • differentiate between speech acts such as assertion, declaration and criticism
Production/Speaking	<ul style="list-style-type: none"> • describe facts and procedures • report observations • articulate and justify hypotheses • present results of individual, partner or group work • use given expressions and phrases

³ vgl. Rahmenlehrplan Jahrgangsstufen 1-10, Teil C Mathematik, S. 31ff, Berlin, Potsdam 2015

⁴ The standards have been taken directly from the Berlin State Curriculum and competencies not addressed in this learning arrangement have been printed in grey.

⁵ vgl. Rahmenlehrplan Jahrgangsstufen 1-10, Teil B Fachübergreifende Kompetenzentwicklung, S. 6-10, Berlin, Potsdam 2015

3.5 References to the general curriculum for media education⁶

Standards of the general media education curriculum	The pupils can
Presentation	<ul style="list-style-type: none"> design a subject and situation appropriate presentation of learning and work results present results of individual and group work to an audience
Communication	<ul style="list-style-type: none"> use the rules of communication

4 Language Development

4.1 Possible language difficulties in task directions

Task	Original text	Alternatives
2	Can you use different ...	Are there other ways ...
3	Decide on an arrangement for your solutions.	Organise your answers. Sort your answers. Group your answers
4	Benjamin claims: ...	Benjamin says: ...
	Each of the multiplication facts can be broken down with only two basic facts.	You only need two rectangles to cover a multiplication fact.
	Is Benjamin correct? Support your answer.	Is Benjamin right? Give reasons.

Pupils must understand the following vocabulary:

poster, trade, known, unknown, squared, array, trace, breakdown,

⁶ vgl. Rahmenlehrplan Jahrgangsstufen 1-10, Teil Fachübergreifende Kompetenzentwicklung, S. 15-22, Berlin, Potsdam 2015

⁶ vgl. Rahmenlehrplan Jahrgangsstufen 1-10, Teil B Fachübergreifende Kompetenzentwicklung, S. 24ff, Berlin, Potsdam 2015

4.2 Vocabulary List for Comprehension

The teacher must be sure that the pupils understand the following mathematical vocabulary.

Nouns	Verbs	Adjectives/Adverbs
times tables	break down	only
multiplication	cover	maximum
basic facts	lay	at the most
rectangle	claim	at least
array	draw/trace	skillful
breakdown	add	next to
squared facts	subtract	on top of
solutions		
possibilities		
addition (plus)		
subtraction (minus)		

4.3 Subject relevant vocabulary and theme specific phrases

During the course of this learning arrangement, the pupils will actively use the following vocabulary and phrases. These will be the foundation for establishing a relevant word list to use when presenting their results.

To break down the multiplication fact, I need..... basic fact rectangles/arrays.

If I lay the rectangles/arrays next to each other, **then** I have to add.

If I cover up part of a rectangle/array, **then** I have to subtract.

We discovered ways to break down this multiplication fact into basic facts.

For a skillful breakdown you need *at the most/maximum/at least/only two basic facts*.

Each multiplication fact can be broken down into ... basic facts.

5 Materials needed for this learning arrangement

Phase / Task	Amount	Material	Preparation/Notes
Introduction / entire learning arrangement	1 set	multiplication fact task cards (extra materials)	copy/print front and back, laminate, cut out
	per pupil	Learning arrangement (LA)	copy alternative: present on a digital whiteboard
Introduction	1	Times Tables Chart (M1)	project on a digital whiteboard alternative: 1x1 poster
	1	Poster „Core facts“(M5)	enlarge
	1 set	Word cards (M6)	copy, cut out
Tasks 1 - 4	2 sets	Basic fact rectangles/arrays (extra materials)	copy/print out, laminate cut out exactly
Tasks 1 und 2	per pupil	Graph paper (M2)	copy alternative: 1x1 cm graph paper
Task 1	as needed	Tip cards (M3)	cut out
Task 3	7	Poster paper (A3)	possibly in the Montessori colours pink, yellow, purple, white, brown and 2x dark blue for the 9 times tables
Task 4	as needed	Tip cards (M4)	cut out

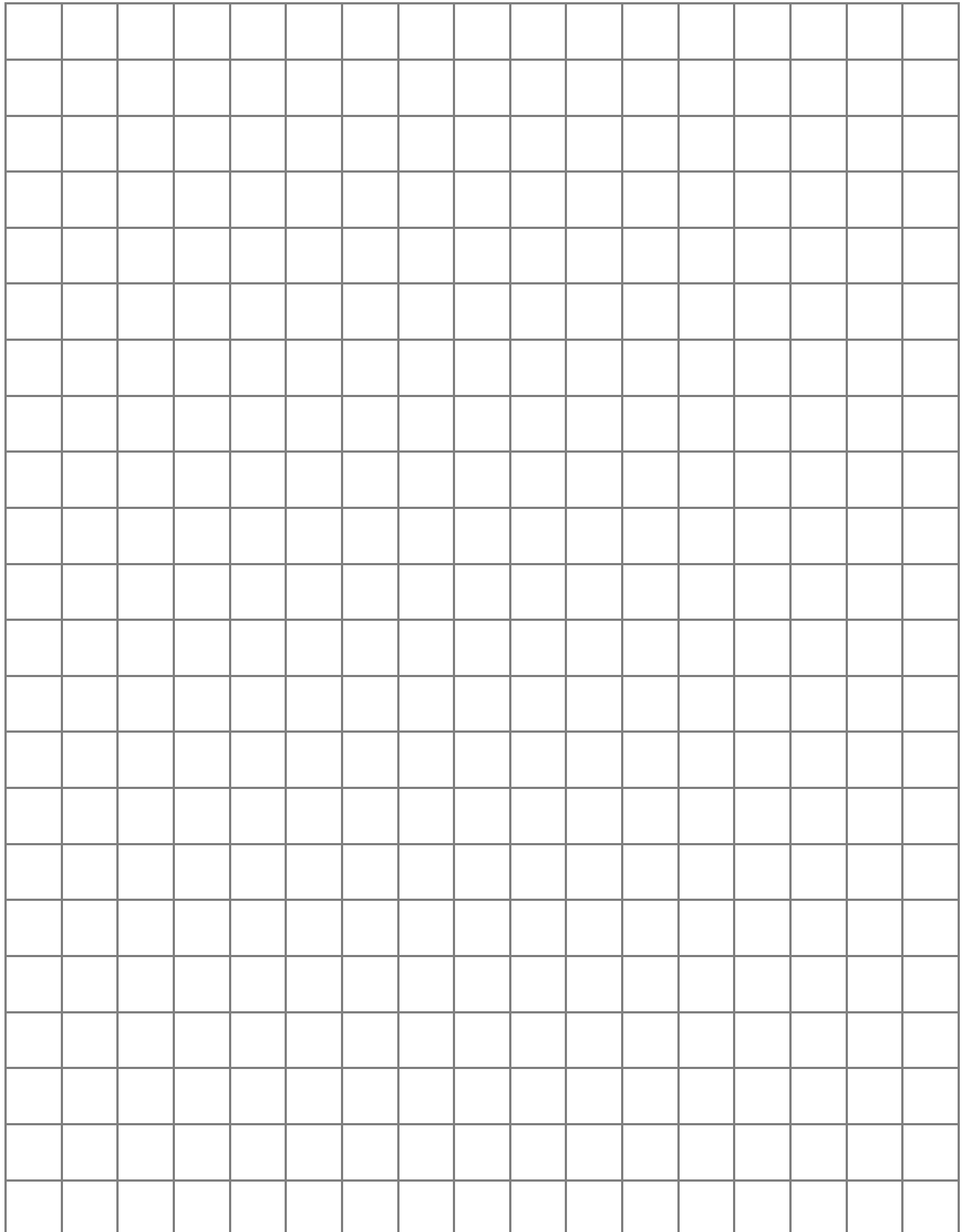
Times Tables Chart (in Montessori colours)

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Times Tables Chart (to colour code / highlight)

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Graph Paper 1 x 1 cm



Tip Card for Task 1

Tip:

$$7 \times 5$$

You can draw like this:
 trace the rectangle/array.

$7 \times 5 = 25 + 10$
 $= 35$

LA 2 / Task 1

Tip:

$$7 \times 5$$

You can draw like this:
 trace the rectangle/array.

$7 \times 5 = 25 + 10$
 $= 35$

LA 2 / Task 1

Tip Cards for Task 4

Tip 1:



Use the grey rectangles/arrays.

LA 2 / Task 4

Tip 2:



Lay a grey rectangle/array on top of the larger basic fact rectangle/array.

LA 2 / Task 4

Tip 1:



Use the grey rectangles/arrays.

LA 2 / Task 4

Tip 2:



Lay a grey rectangle/array on top of the larger basic fact rectangle/array.

LA 2 / Task 4

Basic Facts

(Multiplication)

$$1 \times \square$$

$$2 \times \square$$

$$5 \times \square$$

$$10 \times \square$$

$$\square \times \square$$

Vocabulary cards for a Word Wall

1 times table

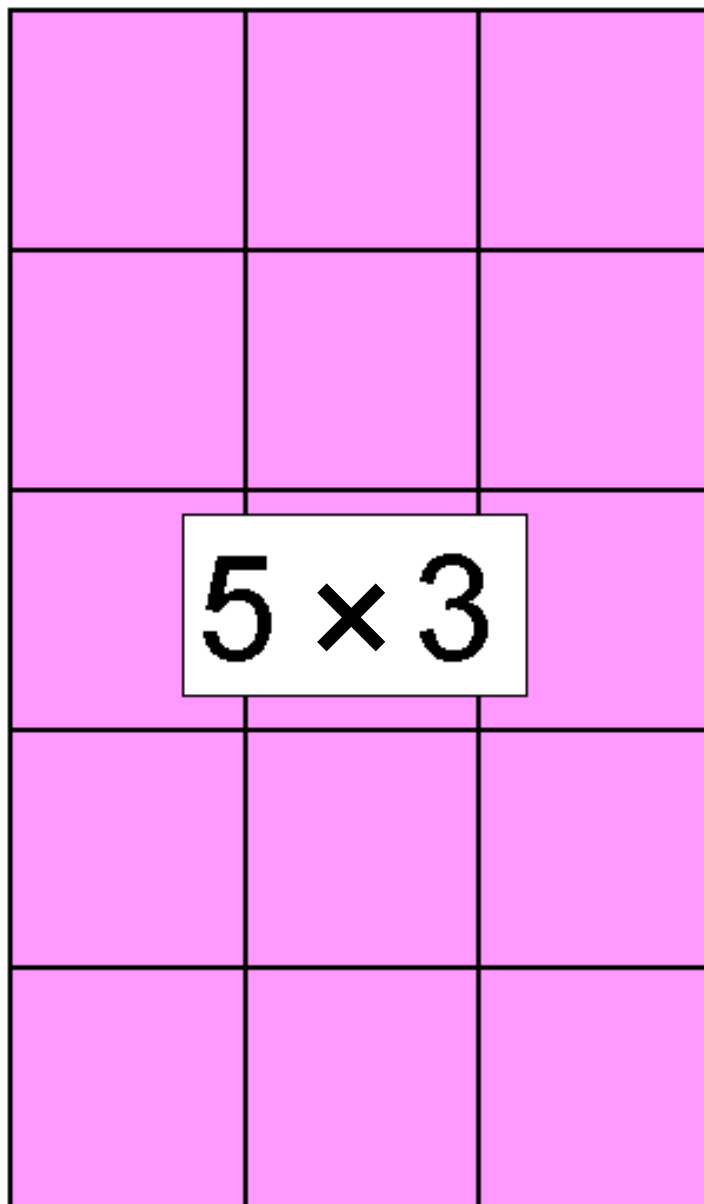
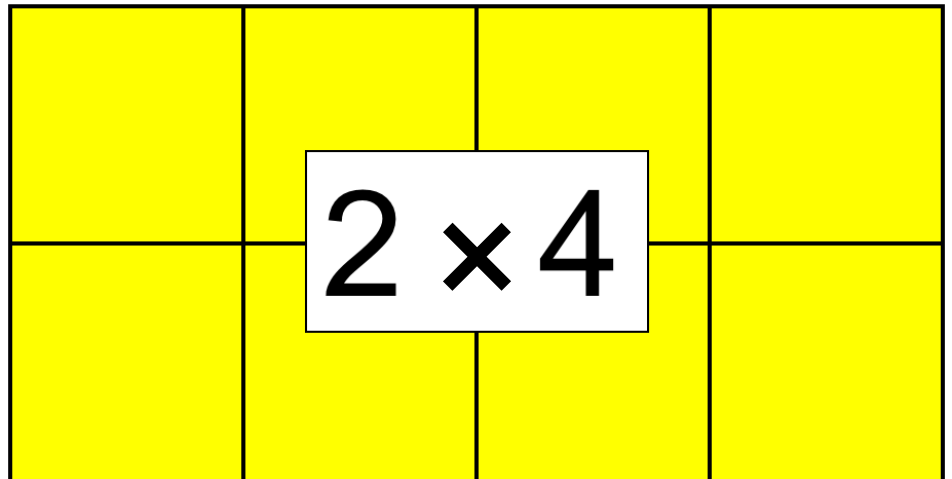
2 times table

5 times table

10 times table

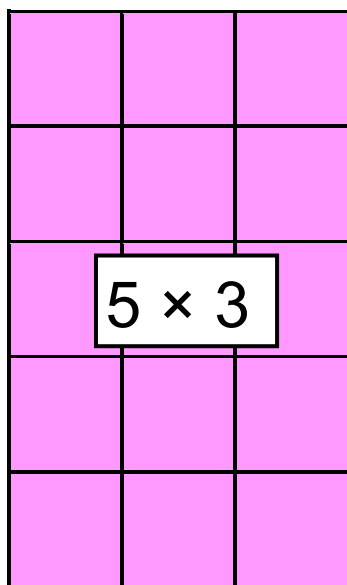
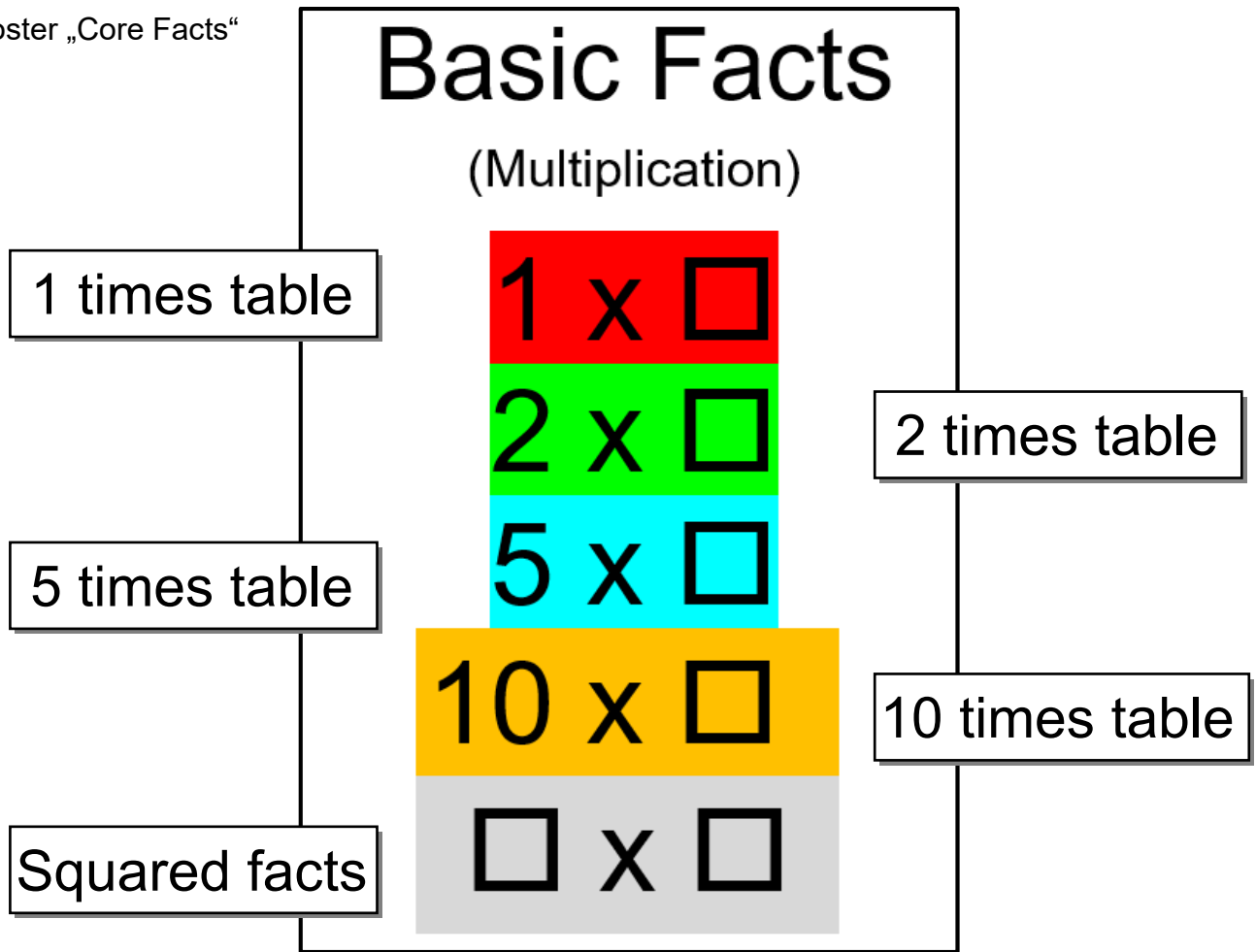
Squared facts

Basic fact rectangles/arrays

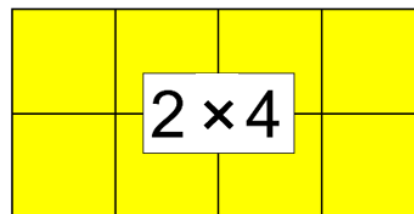


Vocabulary

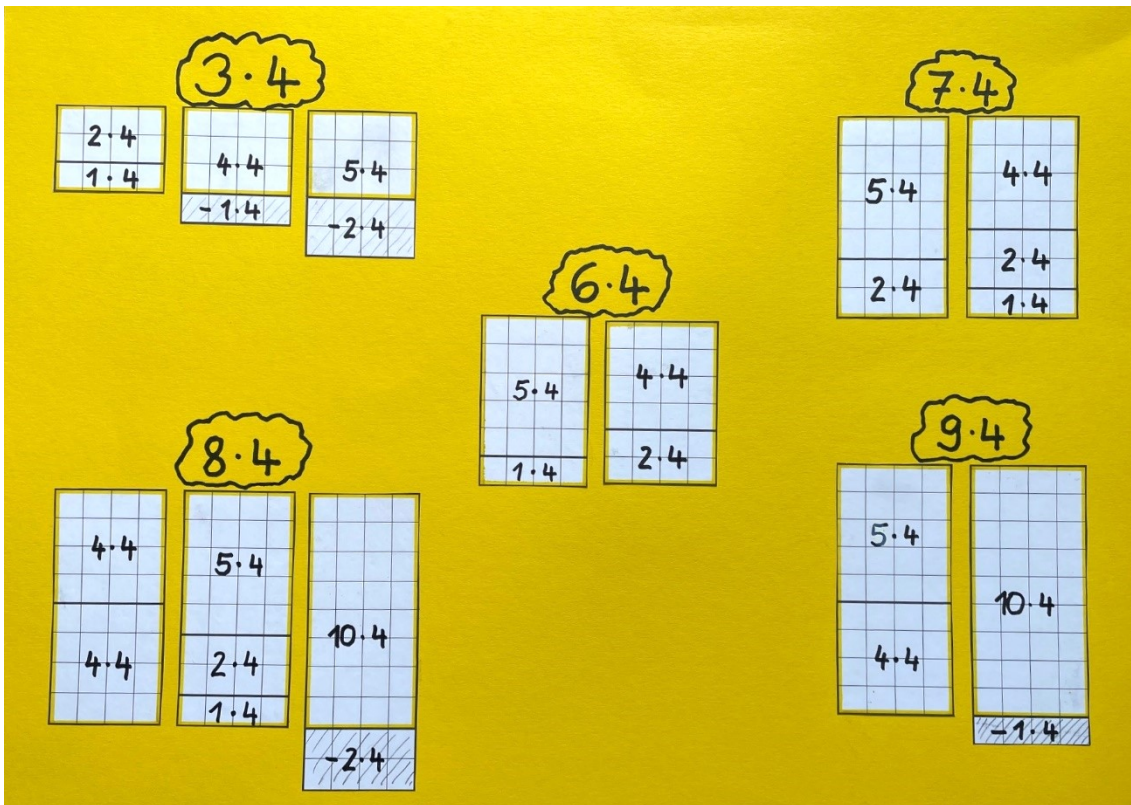
Poster „Core Facts“



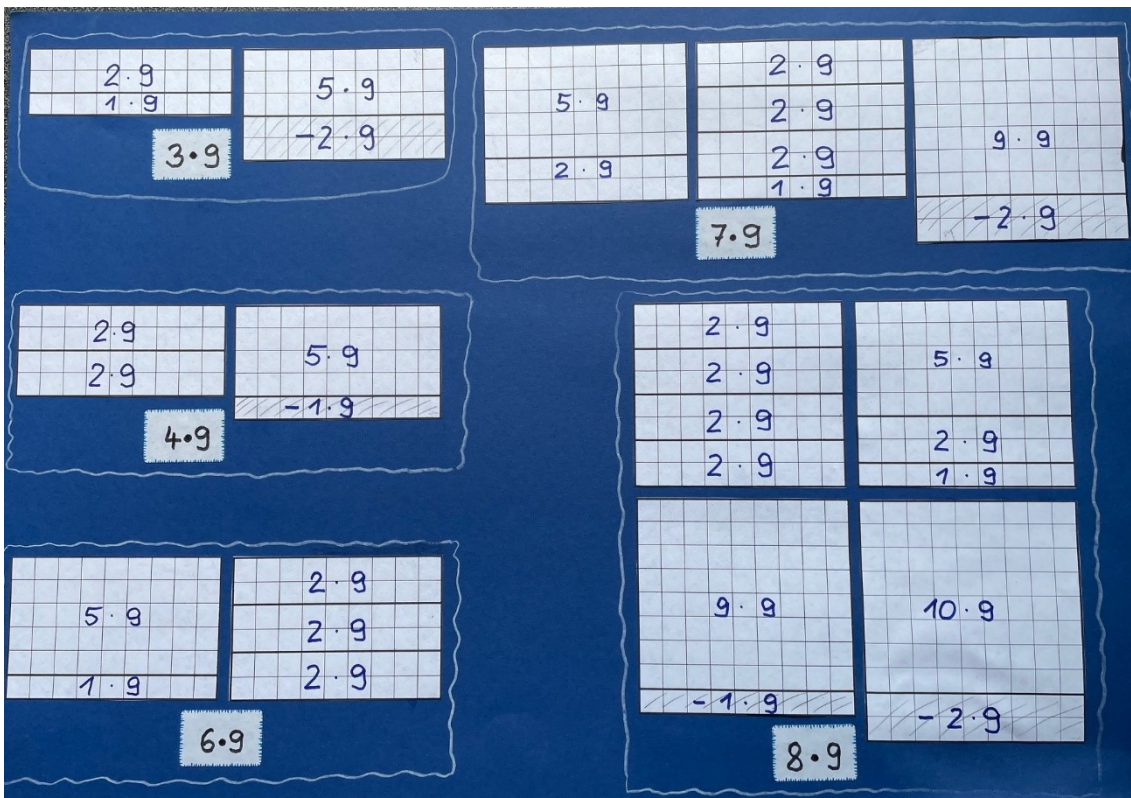
Basic fact rectangles/arrays



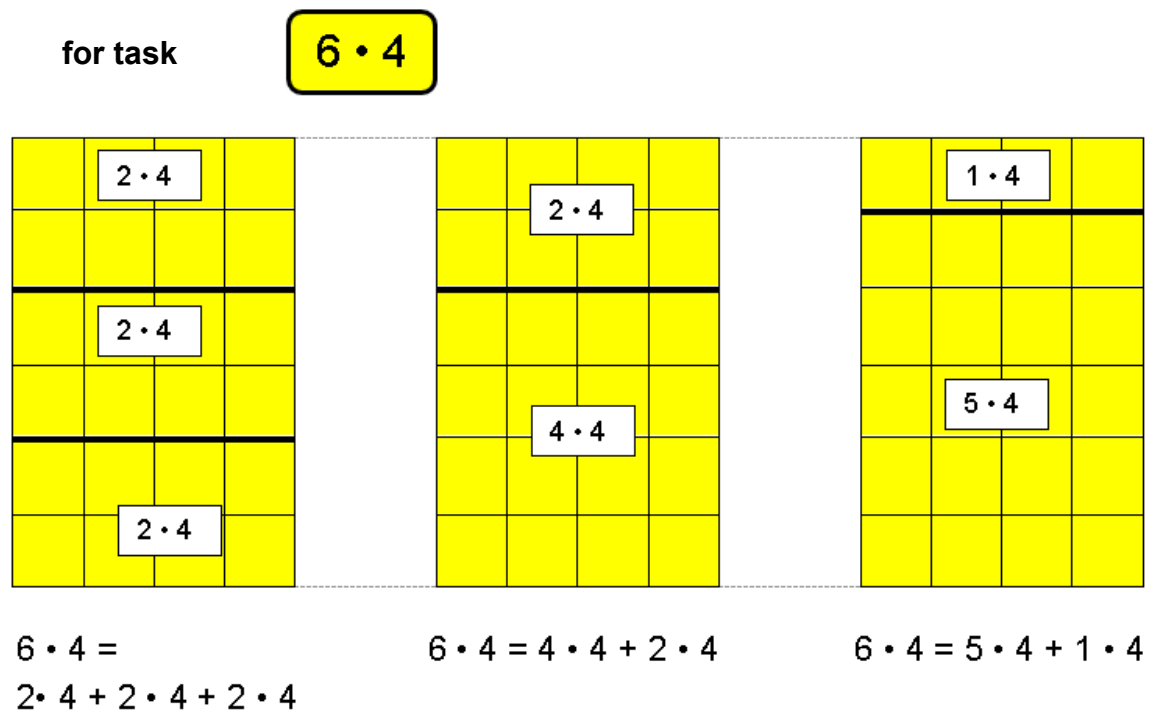
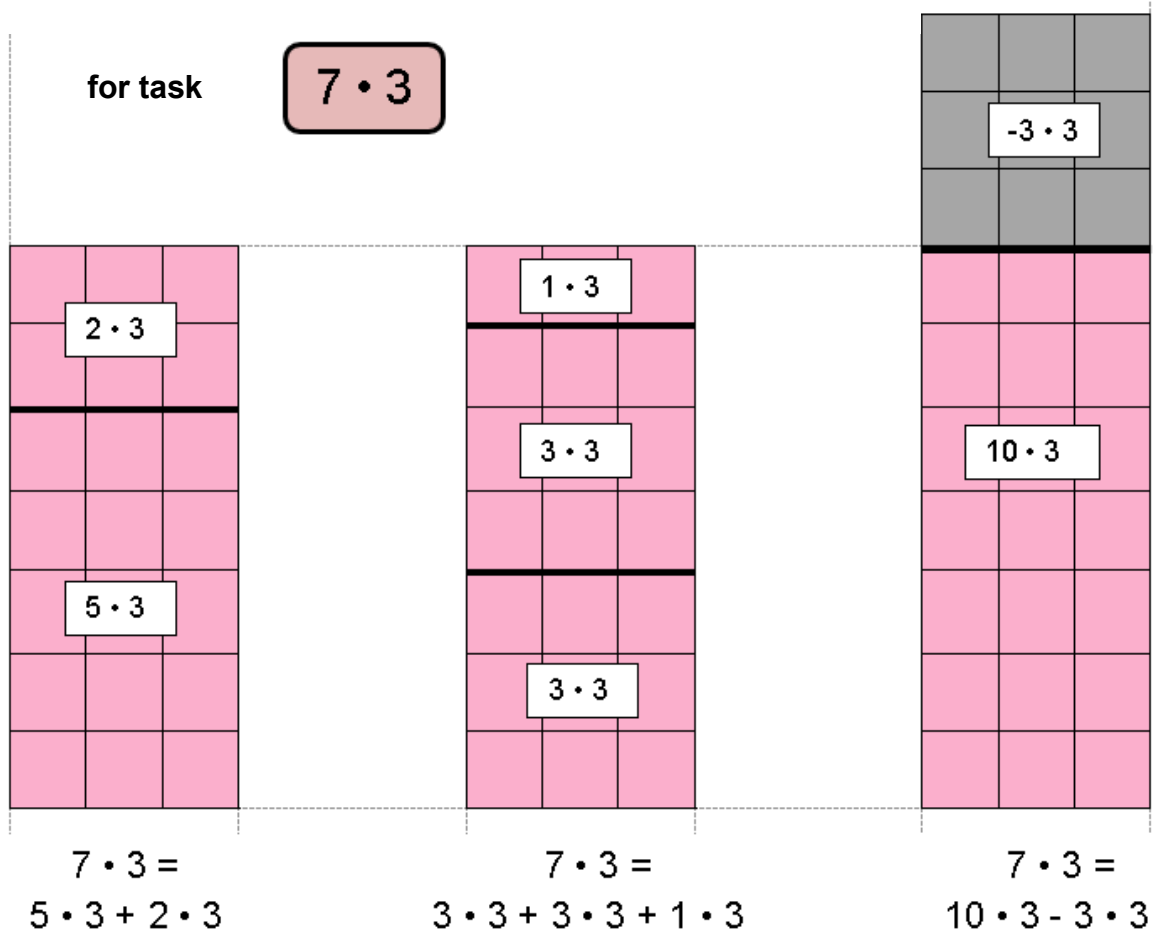
Example of a poster for the 4 times tables

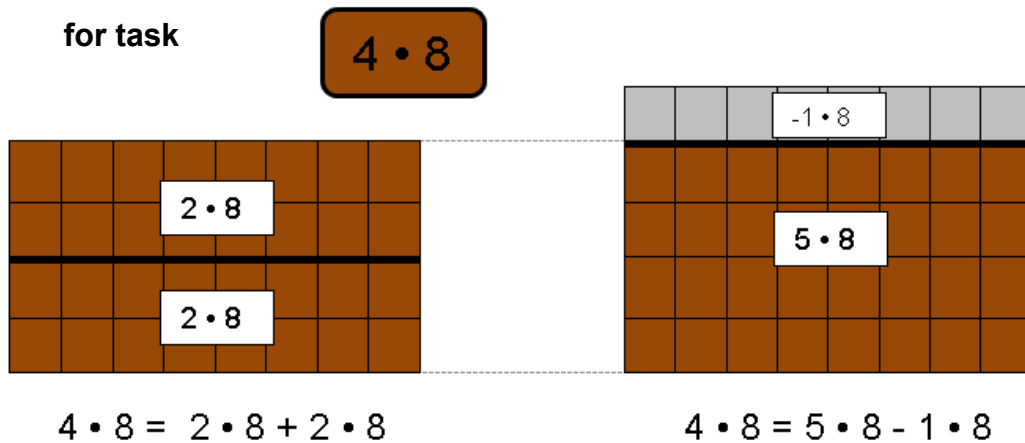
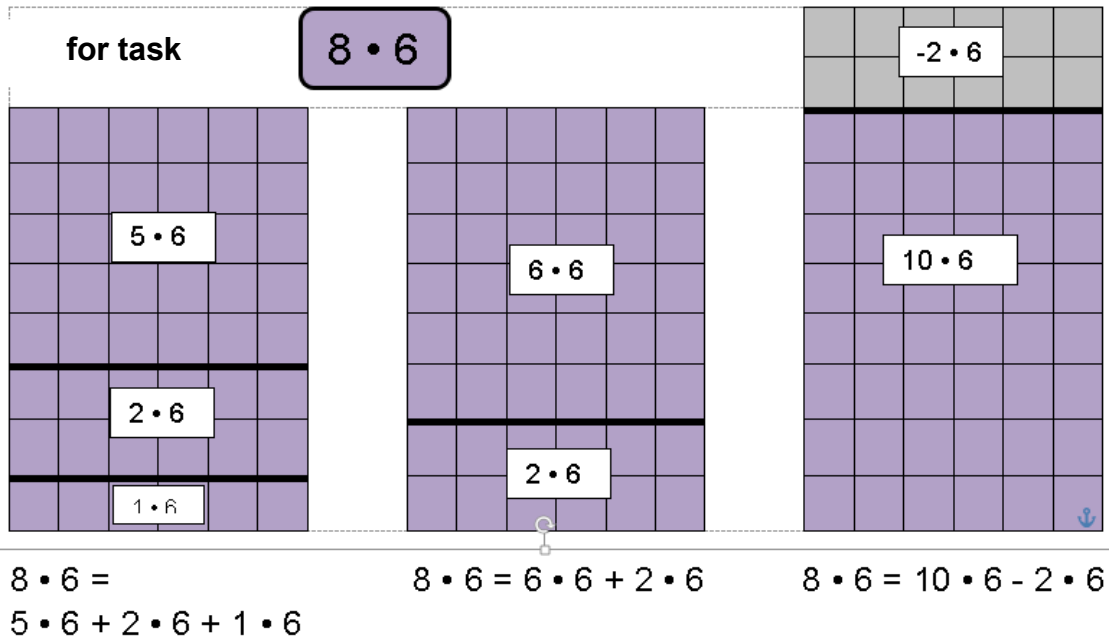


Example of a poster for the 9 times tables



Examples of possible solutions (Task 3)





Generally:

$$\begin{array}{l} 3 \times \square = \\ \hline 2 \times \square = \\ 1 \times \square = \end{array}$$

$$\begin{array}{l} 4 \times \square = \\ \hline 2 \times \square = \\ 2 \times \square = \end{array}$$

$$\begin{array}{l} 6 \times \square = \\ \hline 5 \times \square = \\ 1 \times \square = \end{array}$$

$$\begin{array}{l} 7 \times \square = \\ \hline 5 \times \square = \\ 2 \times \square = \end{array}$$

$$\begin{array}{l} 8 \times \square = \\ \hline 10 \times \square = \\ -2 \times \square = \end{array}$$

$$\begin{array}{l} 9 \times \square = \\ \hline 10 \times \square = \\ -1 \times \square = \end{array}$$

Answer for Task 4

Benjamin is correct because you can break down each multiplication fact from the 1-10 times tables into at the most two facts of 1, 2, 5 or 10.

Sources:

Senatsverwaltung für Bildung, Jugend und Wissenschaft Berlin / Ministerium für Bildung,
 Jugend und Sport des Landes Brandenburg (Hrsg.): Rahmenlehrplan Jahrgangsstufen 1-
 10, Teil B. Berlin, Potsdam 2015

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 Teil C Mathematik, Berlin, Potsdam 2015

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2	Benjamin	Benjamin, Matthia Lux, CC BY SA 4.0 , Lerndokumentation Mathematik-Anregungsmaterialien, S. 35 https://bildung-brandenburg.de/transkigs/fileadmin/user/redakteur/Berlin/Lerndoku_Mathematik_Anregungsmaterialien.pdf [18.02.2021]
2	symbols for individual, partner and group work	Symbole Einzel-, Partner- und Gruppenarbeit, Solveg Schlinske, CC BY SA 4.0 , LU Sehenswürdigkeiten
13,14	light bulb	Glühlampe, Laura Jahn, CC BY SA 4.0 , LU Kernaufgaben
19	Example of a poster for the 4 times tables (Foto)	Beispiel für ein Plakat zur 4er-Reihe (Foto), Susanne Führlich, CC BY SA 4.0 , LU Einmaleins mit Kernaufgaben
19	Example of a poster for the 9 times tables (Foto)	Beispiel für ein Plakat zur 9er-Reihe (Foto), Susanne Führlich, CC BY SA 4.0 , LU Einmaleins mit Kernaufgaben